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LETTER TO THE EDITOR

Effect of anisotropic constraints on self-avoiding walks

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Abstract. Here we study the effect of four types of anisotropic constraints on self-avoiding walks (SAWs) on the square lattice, in which walks along one lattice direction are constrained differently from those of the other three identically constrained lattice directions. By numerical studies we indicate that all these types of anisotropically constrained SAWs belong to the universality class of directed self-avoiding walks.

A self-avoiding walk (SAW) is a random walk which avoids itself such that a self-avoiding walker does not visit a site which he has already visited. In constrained self-avoiding walks further restriction is imposed, which means that the walker in this case is not allowed to visit all the sites which he could visit without constraint.

Recently a considerable amount of interest has been shown in studying the effect of different constraints on SAW statistics, specifically in two dimensions. For example, Grassberger (1982) showed that two-choice SAWs, in which no two successive steps are allowed in the same direction, belong to the same universality class as the ordinary SAW. Directed self-avoiding walks (DSAWs) (Fisher and Sykes 1959, Chakrabarti and Manna 1983) are forbidden to have any step along a specified lattice direction. Field theoretic and exact studies (Cardy 1983, Redner and Majid 1983) show that the critical behaviour of DSAWs is mean field like and anisotropic. In spiral SAWs (Privman 1983), the constraint is such that every step forbids its next step to be in the clockwise (or anticlockwise) direction, so that the walk spirals about a direction perpendicular to the plane of the walk. Such a constraint also leads to a different universality class (Blöte and Hilhorst 1984, Guttmann and Wormald 1984).

At this stage we would like to classify constraints into two categories, isotropic and anisotropic. In isotropically constrained walks, when the walker makes a step along any lattice direction he faces the same restriction in his choice for the next step (e.g. two-choice and spiral SAWs). In anisotropically constrained walks restrictions are different for different lattice directions (e.g. the DSAW).

We consider here four types of anisotropically constrained SAWs on the square lattice which are more non-trivial than the DSAW in the sense that a stronger excluded volume effect is present in these walks. In these constrained SAWs, the restrictions along three lattice directions are the same, but are different for the fourth lattice direction. Specifically they are as follows.

(1) Two-choice restriction along the $+x$ direction and no restriction along the other three directions (figure 1(a)).

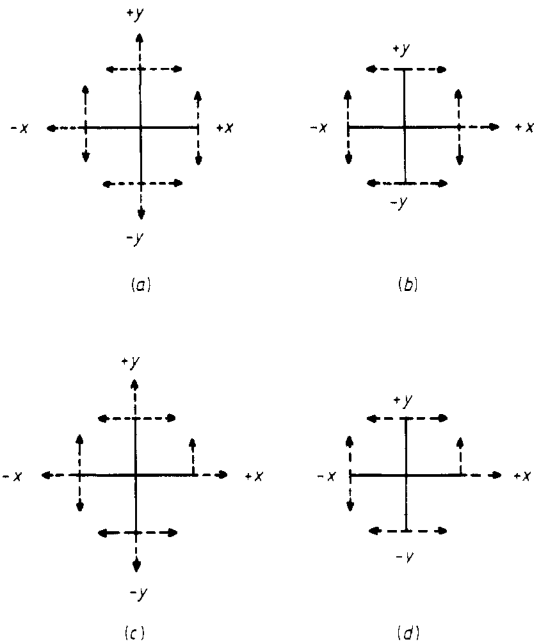

Figure 1. Choices for $(n+1)$th step (shown by broken arrows) for various directions of the nth step (shown by continuous lines): (a), (b), (c) and (d) for types 1, 2, 3 and 4 of constrained SAWs respectively.

(2) No restriction along the $+x$ direction and two-choice restriction along the other three directions (figure 1(b)).

(3) Spiral restriction along the $+x$ direction and no restriction along the other three directions (figure 1(c)).

(4) Spiral restriction along the $+x$ direction and two-choice restriction along the other three directions (figure 1(d)).

Extrapolations of the exact enumeration results for these walks show that these walks belong to the universality class of directed self-avoiding walks.

The total number of independent configurations C_N and the average of the square of the end-to-end distance $\langle R_N^2 \rangle$ for walk length N vary as $C_N \sim \mu^N N^{\gamma-1}$ and $\langle R_N^2 \rangle \sim N^{2\nu}$ in the $N \to \infty$ limit. The average end-to-end distance exponent ν and scaling length exponent γ for SAWs are 0.75 and 43/32 respectively (Nienhuis 1982), whereas for DSAWs the exponents associated with projection of the end-to-end distance vector along and perpendicular to the preferred directions are $\nu_\parallel = 1.0$, $\nu_\perp = 0.5$ and $\gamma = 1.0$ (Cardy 1983). Redner and Majid (1983) introduced a two-choice DSAW in which two perpendicular directions are forbidden for any step. For this walk $\nu_\parallel = \nu_\perp = 1.0$ and $\gamma = 1.0$.

For each kind of anisotropically constrained SAW we first enumerate all the SAW configurations, for a finite walk length N, following Martin (1974). We calculate the number of independent configurations C_N for walk length N, and the average of the square of the projection of the end-to-end distance along the x and y axes (denoted by $\langle R_N^2(x) \rangle$ and $\langle R_N^2(y) \rangle$ respectively) (see table 1). The values of the scaling exponent γ and connectivity constant μ are determined from these simulation results for finite walk length N, following the extrapolation method of Martin *et al* (1967). To find the distance exponent we calculate (Grassberger 1982)

$$\nu_N = (N/2)[(\langle R^2_{N+1}\rangle/\langle R^2_N\rangle) - 1]$$

for both x and y axes. Plotting these values of ν_N against $1/N$ for both x and y axes, we find ν_x and ν_y (for $1/N \to 0$) from separate extrapolations for even and odd values. For different types of anisotropically constrained SAWs, results are given in table 2.

Table 1. C_N, $\langle R^2_N(x)\rangle$ and $\langle R^2_N(y)\rangle$ as defined in the text for types 1, 2, 3, 4 of constrained SAWs respectively.

(*a*)

N	C_N	$\langle R^2_N(x)\rangle$	$\langle R^2_N(y)\rangle$
1	4	0.500 00	0.500 00
2	11	1.090 90	1.454 54
3	31	1.838 70	2.516 12
4	79	2.886 07	4.000 00
5	209	4.023 92	5.473 68
6	535	5.450 46	7.222 42
7	1 393	6.974 15	8.934 67
8	3 559	8.817 08	10.878 33
9	9 191	10.765 85	12.764 87
10	23 467	13.042 82	14.841 94
11	60 299	15.451 94	16.867 84
12	153 923	18.184 07	19.039 15
13	394 457	21.072 21	21.167 03
14	1 006 697	24.280 72	23.408 63
15	2 575 973	27.663 06	25.611 83
16	6 573 319	31.364 26	27.905 54
17	16 805 237	35.252 58	30.164 30

(*b*)

N	C_N	$\langle R^2_N(x)\rangle$	$\langle R^2_N(y)\rangle$
1	4	0.500 00	0.500 00
2	9	1.333 33	0.888 88
3	21	2.333 33	1.333 33
4	41	4.195 12	2.048 78
5	87	6.218 39	2.689 65
6	179	8.916 20	3.396 64
7	377	11.944 29	4.079 57
8	787	15.593 39	4.797 96
9	1 659	19.630 50	5.485 23
10	3 465	24.405 19	6.222 22
11	7 293	29.569 31	6.922 25
12	15 287	35.424 60	7.651 46
13	32 153	41.742 76	8.358 72
14	67 479	48.735 04	9.085 19
15	141 909	56.216 21	9.795 02
16	298 211	64.322 69	10.513 03
17	627 233	72.956 76	11.222 21
18	1318 217	82.235 73	11.940 98
19	2772 623	92.037 98	12.650 65
20	5829 947	102.452 89	13.364 54

Table 1. (continued)

(*c*)

N	C_N	$\langle R_N^2(x \text{ and } y)\rangle$
1	4	0.500 00
2	11	1.363 63
3	30	2.366 66
4	77	3.662 33
5	202	5.014 85
6	516	6.635 65
7	1 338	8.290 73
8	3 413	10.216 23
9	8 794	12.175 68
10	22 437	14.391 27
11	57 614	16.652 39
12	147 043	19.152 83
13	376 884	21.710 26
14	962 144	24.492 68
15	2463 480	27.341 95
16	6290 460	30.403 92

(*d*)

N	C_N	$\langle R_N^2(x)\rangle$	$\langle R_N^2(y)\rangle$
1	4	0.500 00	0.500 00
2	8	1.375 00	0.875 00
3	16	2.437 50	1.312 50
4	27	4.296 29	2.074 07
5	49	6.204 08	2.877 55
6	87	8.574 71	3.885 05
7	157	11.076 43	5.057 32
8	282	13.936 17	6.446 80
9	512	16.890 62	7.992 18
10	916	20.406 11	9.868 99
11	1 658	23.979 49	11.881 78
12	2 990	27.939 79	14.160 53
13	5 412	32.086 50	16.629 89
14	9 761	36.657 41	19.391 14
15	17 668	41.400 61	22.329 97
16	31 928	46.512 90	25.538 33
17	57 793	51.856 19	28.947 38
18	104 496	57.558 71	32.623 87
19	189 171	63.501 24	36.502 34
20	342 164	69.801 48	40.648 18
21	619 397	76.356 71	45.001 56
22	1120 801	83.245 37	49.609 03
23	2028 972	90.408 90	54.433 89
24	3671 659	97.909 84	59.515 30
25	6646 627	105.687 58	64.814 40

From table 2, it is likely that anisotropically constrained SAWS of type 1 and 2 belong to the universality class of DSAWS whereas type 3 and 4 belong to the universality class of two-choice DSAWS.

Table 2. μ, γ, ν_x and ν_y constants for different types of constrained SAWs.

Type	μ	γ	ν_x	ν_y
1	2.535	1.015	1.011	0.527
2	2.107	1.028	0.991	0.489
3	2.550	1.038	0.988	0.988
4	1.811	1.018	1.001	0.997

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